

Naturalness of the Non-Universal MSSM in the light of the recent Higgs results

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Abstract

We analyse the naturalness of the Minimal Supersymmetric Standard Model (MSSM) in the light of recent LHC results from the ATLAS and CMS experiments. We study non-universal boundary conditions for the scalar and the gaugino sector, with fixed relations between some of the soft breaking parameters, and find a significant reduction of fine-tuning for non-universal gaugino masses. For a Higgs mass of about 125 GeV, as observed recently, we find parameter regions with a fine-tuning of $\mathcal{O}(10)$, taking into account experimental and theoretical uncertainties. These regions also survive after comparison with simplified model searches in ATLAS and CMS. For a fine-tuning less than 20 the lightest neutralino is expected to be lighter than about 400 GeV and the lighter stop can be as heavy as 3.5 TeV. On the other hand, the gluino mass is required to be above 1.5 TeV. For non-universal gaugino masses, we discuss which fixed GUT scale ratios can lead to a reduced fine-tuning and find that the recent Higgs results have a strong impact on which ratio is favoured. We also discuss the naturalness of GUT scale Yukawa relations, comparing the non-universal MSSM with the CMSSM.

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1 Introduction

Recently, the LHC experiments ATLAS and CMS have reported the discovery of a new resonance, mainly based on excesses of events in the $\gamma\gamma$ and ZZ^* channels. The combined significance amounts about 5σ for both, ATLAS and CMS [1]. The new particle is compatible with the Standard Model (SM) Higgs boson with a mass of $m_h \approx 125 \div 126$ GeV [2]. In the context of low-energy Supersymmetry (SUSY), such a value still lies in the predicted range of the Minimal Supersymmetric Standard Model (MSSM), provided quite heavy top squarks (with masses $\gtrsim \mathcal{O}(1)$ TeV) and/or large left-right stop mixing (see for instance [3]) are present. Regarding direct searches for SUSY particles [4], based on $\sim 5 \text{ fb}^{-1}$ of data, only negative results have been reported.

From the point of view of the MSSM, the negative results of direct SUSY searches and the comparatively heavy Higgs mass are well consistent with each other. However, as the main motivation for SUSY is the solution to the hierarchy problem, which naively requires SUSY masses of the order of the electroweak symmetry breaking scale, one may ask whether the MSSM can still accomplish this task in a natural way (or at least for a moderate fine-tuning price). Regarding specific MSSM scenarios, it turns out that in the constrained MSSM (CMSSM) a significant amount of fine-tuning (at least $\gtrsim \mathcal{O}(100)$) is unavoidable to explain a Higgs mass of $m_h \approx 125 \div 126$ GeV. Following our paper [5] on the CMSSM, we thus find it interesting to re-address the naturalness of the MSSM in the context of models where certain universality assumptions on the SUSY parameters are relaxed (e.g. for scalar and gaugino masses).

In order to envisage the most promising ways to reduce the fine-tuning, we are going to adopt the following strategy: we start with considering a generic setup with 17 independent parameters inspired by the phenomenological MSSM (pMSSM), in order to identify those giving the dominant contributions to the fine-tuning. This allows us to identify the rigid relations among the parameters which can decrease fine-tuning. After discussing several possibilities, we will mainly focus on certain non-universal relations among gaugino masses, whose possible impact on fine-tuning have been studied before the LHC results by [6, 7].⁶ In this context, we will study the interplay between the fine-tuning, the model predictions for the light Higgs mass and the Grand Unified Theory (GUT) scale ratios of the the gaugino masses and of the third family Yukawa couplings, which are key quantities for discriminating among different SUSY (and SUSY GUT) models. In our analysis, we include various relevant experimental constraints, e.g. from $\text{BR}(b \rightarrow s\gamma)$, $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ and $\text{BR}(B_u \rightarrow \tau\nu_\tau)$. We do not impose strict constraints from requiring that the $(g-2)_\mu$ deviation from the SM (currently at the level of 3.2σ [11]) is explained by SUSY.

The rest of the paper is organized as follows: in the next section we study in a semi-analytic way the dependence of the electroweak scale on GUT scale boundary conditions whose choice is inspired by the pMSSM. Thanks to this analysis we can identify regions with low fine-tuning for certain limiting cases and identify as most promising case non-universal gaugino masses. In section 3 we revisit briefly the dependence of GUT scale Yukawa coupling ratios on low energy supersymmetric threshold corrections. Thereafter we make an extensive numerical analysis of the fine-tuning in an MSSM scenario with non-universal gaugino masses at the GUT scale and compare the results to various experimental results, most importantly

⁶A recent discussion of the fine-tuning price of a 125 GeV Higgs after the first hints in December 2011 within several SUSY models has been also given in [8] (see also [9, 10]).

the recent discovery of a new resonance around 125 GeV compatible with a Higgs boson. In the final section 5 we summarize and conclude.

2 Fine-Tuning in the MSSM

Fine-tuning in the MSSM has been extensively discussed in the literature starting with [12].⁷ As we are going to follow the same approach as in our recent paper [5], we give here just a brief summary of the most important points.

In the MSSM, by minimizing the scalar potential, the Z -boson mass can be computed in terms of the supersymmetric Higgs parameter μ and the soft SUSY breaking mass terms of the up- and down-type Higgs doublets. For moderate and large $\tan\beta$, one finds at tree level

$$\frac{M_Z^2}{2} = -|\mu|^2 - m_{H_u}^2 + \mathcal{O}(m_{H_{u,d}}^2/(\tan\beta)^2). \quad (1)$$

The value of the low energy observable M_Z can thus be obtained in terms of the fundamental parameters of the high-energy theory, by considering the renormalization group (RG) evolution of μ and m_{H_u} , which are assumed to be both of the order of the SUSY breaking mass scale, from the GUT scale down to low energy. Since experimental constraints force the SUSY scale to be somehow larger than M_Z , a certain amount of tuning is needed. In order to quantify fine-tuning, the following measure has been introduced [12]

$$\Delta_a = \left| \frac{\partial \log M_Z}{\partial \log a} \right| = \left| \frac{a}{2M_Z^2} \frac{\partial M_Z^2}{\partial a} \right|. \quad (2)$$

Δ_a reflects the dependence of M_Z on the variation of a given GUT-scale Lagrangian parameter a . This definition not only encompasses the obvious fine-tuning in μ which is needed to fulfill Eq. (1), but it also covers the tuning needed to have a small $m_{H_u}^2$, while other soft SUSY breaking parameters entering its RG evolution are relatively large. The overall measure of fine-tuning for a given parameter point is then defined as the maximum of all the single Δ_a 's:

$$\Delta = \max_a \Delta_a. \quad (3)$$

Turning to the contributions Δ_a of the single parameters, let us first notice that the fine-tuning in μ is rather special as its low-energy value is determined by imposing the correct M_Z . Hence, setting aside RG effects for the moment, Δ_μ can be approximately expressed in terms of $m_{H_u}^2$ as

$$\Delta_\mu \approx \left| 2 \frac{m_{H_u}^2}{M_Z^2} + 1 \right|. \quad (4)$$

In the numerical analysis, the running of μ is of course included. It turns out that in the constrained MSSM Δ_μ is often the dominant term in Δ followed by the tuning in A_0 , m_0 and $M_{1/2}$. Performing a scan of the CMSSM parameter space, we found in [5] that the fine-tuning in μ dominates in 49(74)% of the cases, A_0 in 29(0)%, m_0 in 11(0)% and $M_{1/2}$ in 11(26)% for the $(g-2)_\mu$ constraint applied at $5(2)\sigma$.

The discussion of fine-tuning for the other parameters is more involved as they enter Eq. (1) only via the RG evolution of $m_{H_{u,d}}^2$. It is, however, possible to approximately express $m_{H_u}^2$ as a polynomial in terms of the high-energy parameters of the theory.

⁷For an extensive list of references, see for instance [8, 13].

In the following we will give approximate formulas for the dependence of $m_{H_u}^2$ on the assumed fundamental GUT-scale parameters at low energies, which can tell us for which choice of parameters we can expect a reduced fine-tuning.

Before we come to this we like to note that fine-tuning measures should be taken with some caution. It is a matter of individual taste how much fine-tuning one accepts as “natural” and it also depends on the fine-tuning definition used. Thus we want to stress that our aim here is mainly to compare different parameter regions and only the relative fine-tuning difference between them would render a region more attractive to us.

2.1 Fine-Tuning à la pMSSM

To express $m_{H_u}^2$ in terms of high-energy parameters, our choice of the independent parameters is inspired by the so-called phenomenological MSSM (pMSSM) [14] (but with the parameters defined at the GUT scale). In particular, we consider different scalar masses for the chiral superfields of the MSSM. The first two generation sfermions are assumed to be degenerate, while we allow a splitting for the third generation. The gaugino masses are taken to be non-universal. Concerning the trilinears we introduce three independent parameters: one for the up squarks, one for the down squarks and one for the charged sleptons. The main difference to the usual pMSSM is in the Higgs sector. Namely, instead of low scale μ and the CP-odd Higgs mass m_{A_0} , we take the GUT scale value of $m_{H_u}^2$ and $m_{H_d}^2$ to be the free parameters and denote them by $m_{h_u}^2$ and $m_{h_d}^2$ respectively.

The dependence of $m_{H_u}^2$ on the GUT scale parameters can be deduced from its RGE (see e.g. [15]) and can be written as

$$m_{H_u}^2 = \sum_i a_i m_i^2 + \frac{1}{2} \sum_{i,j} N_i b_{ij} N_j, \quad (5)$$

where the $N_i \equiv (M_1, M_2, M_3, A_t, A_b, A_\tau)$ are assumed to be real and the matrix b_{ij} is a general symmetric 6×6 matrix.

To make the deviation from the universality assumption of the CMSSM more evident we introduce the following dimensionless quantities:

$$\eta_\alpha = M_\alpha/M_3 \quad \text{with } \alpha = 1, 2, 3 \quad \eta_i = A_i/M_3 \quad \text{with } i = t, b, \tau. \quad (6)$$

Note that the convention we use here and in the rest of the paper implies $\eta_3 = 1$ and we can recast the N_i as

$$N_i = M_3 \cdot (\eta_1, \eta_2, \eta_3, \eta_t, \eta_b, \eta_\tau). \quad (7)$$

The fine-tuning measure introduced in Eq. (3) can now be easily expressed as

$$\Delta_{m_i^2} = \left| 2a_i \frac{m_i^2}{M_Z^2} \right|, \quad \Delta_m = \max_i \Delta_{m_i^2}, \quad (8)$$

for the scalar masses, whereas for the other soft terms we find

$$\Delta_{N_i} = \left| \frac{\sum_j N_i b_{ij} N_j}{M_Z^2} \right| \quad (\text{no sum over } i), \quad \Delta_N = \max_i \Delta_{N_i}, \quad (9)$$

and

$$\Delta_\mu \approx \left| 2 \frac{\sum_i a_i m_i^2 + \frac{1}{2} \sum_{i,j} N_i b_{ij} N_j}{M_Z^2} + 1 \right|. \quad (10)$$

After this general introduction to fine-tuning à la pMSSM, we turn to expressing $m_{H_u}^2$ at the SUSY scale for a point which corresponds to $M_3 = 0.6$ TeV, $m_i = 1.5$ TeV for every i , $\eta_i = N_i/M_3 = (1, 1, 1, -5, -5, -5)$, $\tan \beta = 30$ and μ positive. This point is in agreement with the Higgs mass range quoted [1] and the $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ bound [16] and we find

$$\begin{aligned}
m_{H_u}^2(M_{\text{SUSY}}) = & -0.0459m_{\tilde{Q}_1}^2 + 0.0988m_{\tilde{U}_1}^2 - 0.0469m_{\tilde{D}_1}^2 + 0.0488m_{\tilde{L}_1}^2 - 0.0541m_{\tilde{E}_1}^2 \\
& - 0.3347m_{\tilde{Q}_3}^2 - 0.2500m_{\tilde{U}_3}^2 - 0.0154m_{\tilde{D}_3}^2 + 0.0245m_{\tilde{L}_3}^2 - 0.0236m_{\tilde{E}_3}^2 \\
& + 0.6481m_{\tilde{h}_u}^2 + 0.0273m_{\tilde{h}_d}^2 \\
& - M_3^2(1.2865 - 0.0216\eta_1 - 0.0242\eta_1^2 + 0.0230\eta_2 - 0.2177\eta_2^2 + 0.0813\eta_1\eta_2) \\
& + M_3^2(0.2521\eta_t + 0.0208\eta_b + 0.0175\eta_\tau) \\
& + M_3^2\eta_1(0.0087\eta_t - 0.0028\eta_b + 0.0053\eta_\tau) \\
& + M_3^2\eta_2(0.0852\eta_t - 0.0086\eta_b - 0.0114\eta_\tau) \\
& + M_3^2(0.0022\eta_b^2 - 0.1244\eta_t^2 - 0.0001\eta_\tau^2) \\
& + M_3^2(0.0068\eta_b\eta_t + 0.0017\eta_b\eta_\tau + 0.0007\eta_t\eta_\tau) ,
\end{aligned} \tag{11}$$

where η_3 is set to be 1. One can already spot here the most important contributions to come from the stops, the gluinos and the Higgs sector itself. The coefficients a_i appearing in Eq. (5) can be directly read off from Eq. (11). In order to show the correlations among the parameters $\eta_i = N_i/M_3$ in a transparent way, we display the coefficients b_{ij} as a symmetric matrix:

$$b_{ij} = \begin{pmatrix} 0.0242 & -0.0813 & 0.0216 & 0.0087 & -0.0028 & 0.0053 \\ & 0.2177 & -0.0230 & 0.0852 & -0.0086 & -0.0114 \\ & & -1.2865 & 0.2521 & 0.0208 & 0.0175 \\ & & & -0.1244 & 0.0068 & 0.0007 \\ & & & & 0.0022 & 0.0017 \\ & & & & & -0.0001 \end{pmatrix} . \tag{12}$$

Even though the coefficients a_i and b_{ij} were numerically obtained in a specific point of the parameter space, we checked that they provide a reasonably accurate estimate of $m_{H_u}^2$ in wide regions of parameter space. The corresponding uncertainty of the coefficients is estimated to be of the order of 10-20%. This is good enough for the qualitative discussion we present in the following subsections on possible strategies to reduce the fine-tuning. Nevertheless, the results we present in section 4 are based on a full numerical analysis, with the RGE evolution of all parameters computed in each point of the parameter space.

2.2 Our Strategy

In the attempt to find SUSY models with reduced fine-tuning we assume that the underlying model of SUSY breaking predicts certain fixed relations between the SUSY breaking parameters at high energy (e.g. at the GUT scale). As a consequence, cancellations among different contributions in Eq. (11) can occur which lead to a reduced fine-tuning. We note that such a behaviour is a well-known property of models with universal scalar masses (like the CMSSM), for which the coefficients a_i in Eq. (11) cancel almost completely.⁸ However, in the

⁸This property is known under the name of “focus point” [17], for a recent quantitative discussion see, e.g. [5].

light of the recent Higgs results the CMSSM unavoidably has a certain amount of fine-tuning either from quite heavy squarks or from a large A -term (in maximal mixing scenarios). In this paper, we therefore consider the possibility that the underlying theory predicts different, non-universal (but fixed) boundary conditions at the GUT scale, namely non-universal scalar masses (NUSM) and non-universal gaugino masses (NUGM). Although such relations may not hold exactly in realistic models, they may guide towards more natural SUSY scenarios.

2.3 Fine-Tuning from the Scalar Sector

We now discuss possible NUSM scenarios. To start with, let us note that the situation of universal sfermion masses in the CMSSM features a significant reduction of fine-tuning, since an almost-complete cancellation occurs automatically for the largest contributions in Eq. (11), i.e. between the term with $m_{h_u}^2$ and the terms with $m_{\tilde{Q}_3}^2$ and $m_{\tilde{U}_3}^2$. From the point of view of naturalness, such a fixed relation between $m_{h_u}^2$ and $m_{\tilde{Q}_3}^2, m_{\tilde{U}_3}^2$ is desirable.

Let us now consider non-universal fixed ratios for the soft scalar masses motivated by unified theories: if we assume that at the GUT scale the gauge interactions unify to one single interaction then the soft masses for the various fields are not independent anymore because they are (partially) unified in common representations. Two very prominent paths to unification are $SU(5)$ [18] and Pati–Salam [19], which can both be embedded in $SO(10)$ [20]. For simplicity we assume that possible higher order GUT symmetry breaking corrections, which might induce splittings within one representation are negligibly small and we remind that we have set the first two generations to have the same soft SUSY breaking masses.

- We begin with the case of $SU(5)$, where we find for $m_{H_u}^2$

$$\begin{aligned} m_{H_u}^2(M_{\text{SUSY}}) = & -0.0012m_{\tilde{T}_1}^2 + 0.0019m_{\tilde{F}_1}^2 - 0.6083m_{\tilde{T}_3}^2 + 0.0091m_{\tilde{F}_3}^2 \\ & + 0.6481m_{h_u}^2 + 0.0273m_{h_d}^2 \\ & + \text{gaugino masses and trilinear terms} , \end{aligned} \quad (13)$$

where $m_{\tilde{T}_i}^2$ stands for the masses of the tenplet of $SU(5)$, which contains the coloured doublet, the up-type coloured singlet and charged leptonic singlet ($m_{\tilde{Q}_i}^2 = m_{\tilde{U}_i}^2 = m_{\tilde{E}_i}^2 = m_{\tilde{T}_i}^2$), and $m_{\tilde{F}_i}^2$ for the fiveplet of $SU(5)$, which contains the down-type coloured singlet and the leptonic doublet ($m_{\tilde{D}_i}^2 = m_{\tilde{L}_i}^2 = m_{\tilde{F}_i}^2$). We allow an arbitrary splitting from the Higgs fields from the other scalar mass parameters and also among each other. One can clearly see that in order not to strongly increase the fine-tuning one has to impose a fixed relation where $m_{\tilde{T}_3}^2 \approx m_{h_u}^2$ holds, otherwise one has to consider the fine-tuning from each parameter separately, cf. Eq. (3).

- We come now to the Pati–Salam case for which we find

$$\begin{aligned} m_{H_u}^2(M_{\text{SUSY}}) = & 0.0029m_{\tilde{l}_1}^2 - 0.0022m_{\tilde{\tau}_1}^2 - 0.3102m_{\tilde{l}_3}^2 - 0.2890m_{\tilde{\tau}_3}^2 + 0.6754m_h^2 \\ & + \text{gaugino masses and trilinear terms} , \end{aligned} \quad (14)$$

where in this case \tilde{l} denotes the left-handed doublets ($m_{\tilde{Q}_1}^2 = m_{\tilde{L}_i}^2 = m_{\tilde{l}_i}^2$), $\tilde{\tau}$ denotes the right-handed doublets ($m_{\tilde{U}_1}^2 = m_{\tilde{D}_i}^2 = m_{\tilde{E}_1}^2 = m_{\tilde{\tau}_i}^2$) and h denotes the Higgs bi-doublet ($m_{h_u}^2 = m_{h_d}^2 = m_h^2$). The conclusions are similar to the $SU(5)$ case. The fixed relation one now has to impose is $m_{\tilde{l}_3}^2 \approx m_{\tilde{\tau}_3}^2 \approx m_h^2$.

- Finally, let us discuss the situation in $SO(10)$ GUTs. Here one considers the soft mass terms for the matter fields $m_{16_1}^2$ and $m_{16_3}^2$ and for the Higgs fields $m_{h_u}^2 = m_{h_d}^2 = m_{10}^2$ (with possible $m_{h_u}^2 \neq m_{h_d}^2$ from D-term splitting), obtaining

$$m_{H_u}^2(M_{\text{SUSY}}) = -0.0031m_{16_1}^2 - 0.5992m_{16_3}^2 + 0.6754m_{10}^2 \quad (15)$$

+ gaugino masses, trilinear terms and D-term splitting terms ,

which implies that again, a fixed relation beyond the ones from the GUT itself would have to be imposed (between m_{10}^2 and $m_{16_3}^2$) in order to strongly decrease fine-tuning.

A few additional comments are in order. To start with, one can see that the soft terms for the first two families enter with small coefficients into $m_{H_u}^2$. This illustrates the well-known fact that these soft masses can be significantly larger than the third family ones, without paying a large fine-tuning price. Furthermore, in the above discussion we have focused on fixed relations from GUT structures. As we have already noted, with these fixed relations alone one would increase fine-tuning compared to the CMSSM case. From the above equations one can imagine additional fixed relations on top of these structures which could in principle reduce fine-tuning by effectively leading to a cancellation in the contributions to $m_{H_u}^2$. However, we will not investigate such cases in more detail here.

Another relevant remark is that we have only discussed here “direct” effects on the fine-tuning, i.e. the effects on the fine-tuning measure for a fixed SUSY parameter point. We note that there are also “indirect” effects expected, i.e. when introducing a non-universality allows to avoid certain constraints on the SUSY parameter space and makes regions with lower fine-tuning accessible (e.g. a “compressed” spectrum helps to evade LHC constraints).

Finally, let us notice that small splittings in models where the soft parameters are universal (CMSSM-like) at the leading order and get only corrected by small additional contributions (e.g. by higher dimensional operators in flavour models, see, for instance, [21]) can have a certain degree of non-universality without increasing the fine-tuning too much. However, large non-universalities ($\mathcal{O}(1)$) again lead back to the same situation as in the general case à la pMSSM.

In summary, we conclude that it seems unavoidable to have the scalar mass parameters – at least the high energy stop and Higgs soft masses – almost degenerate to keep the fine-tuning small, like for instance in the CMSSM. Non-universal scalar masses tend to make the fine-tuning worse rather than improving the situation. We will therefore not study this case numerically in more detail.

2.4 Fine-Tuning from Gaugino Masses and Trilinear Couplings

Now we turn to the gaugino masses and the trilinear SUSY breaking couplings. They appear together in the $m_{H_u}^2$ polynomial and hence should not be discussed independently. For the sake of simplicity we assume here that the trilinear couplings are universal $\eta_t = \eta_b = \eta_\tau = \eta_A$.⁹

⁹ In principle, one can reduce the fine-tuning as well by assuming definite relations between η_t , η_b , and η_τ just as we will demonstrate here for the gaugino mass parameters. However, the coefficients involving η_b and η_τ in Eq. (11) are very small and hence the values of η_b and η_τ should be very large.

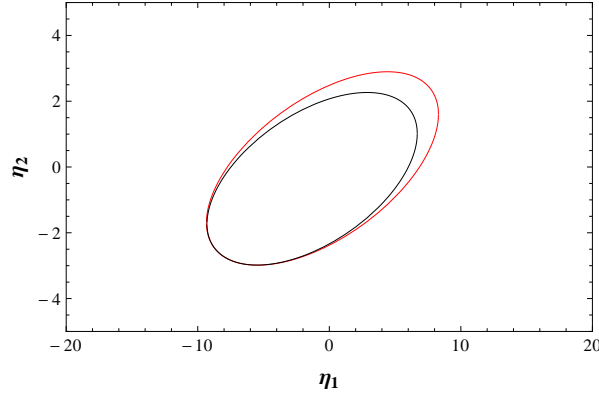


Figure 1: The two ellipses in the η_1 - η_2 plane which minimise the fine-tuning. The red ellipse corresponds to Eq. (18) and the black one to Eq. (19).

In this case the $m_{H_u}^2$ polynomial reads

$$\begin{aligned}
m_{H_u}^2(M_{\text{SUSY}}) &= -M_3^2(1.2865 - 0.0216\eta_1 - 0.0242\eta_1^2 + 0.0230\eta_2 - 0.2177\eta_2^2 + 0.0813\eta_1\eta_2) \\
&\quad + M_3^2\eta_A(0.2904 + 0.0112\eta_1 + 0.0652\eta_2) - 0.1131M_3^2\eta_A^2 \\
&\quad + \text{scalar masses} \\
&\equiv (f_1(\eta_1, \eta_2) + f_2(\eta_1, \eta_2)\eta_A + f_3\eta_A^2)M_3^2 + \dots,
\end{aligned} \tag{16}$$

where we have again set $\eta_3 = 1$. Now the question arises for which values of the parameters the fine-tuning, as defined in Eqs. (8) and (10), is minimised.

The overall fine-tuning, considering only this sector is

$$\Delta \approx \left| \frac{M_3^2}{M_Z^2} \right| \max \left\{ \begin{array}{l} |2f_3\eta_A^2 + f_2(\eta_1, \eta_2)\eta_A|, \\ |2f_1(\eta_1, \eta_2) + f_2(\eta_1, \eta_2)\eta_A| \end{array} \right\}. \tag{17}$$

Setting these two equations to zero we find two possible solutions:

$$\textbf{Solution 1: } f_1(\eta_1, \eta_2) = 0, \quad \eta_A = 0, \tag{18}$$

$$\textbf{Solution 2: } f_1(\eta_1, \eta_2) = \frac{f_2(\eta_1, \eta_2)^2}{4f_3}, \quad \eta_A = -\frac{f_2(\eta_1, \eta_2)}{2f_3}. \tag{19}$$

With the parameters in Eq. (16), the two solutions define two different ellipses in the η_1 - η_2 plane, see Fig. 1. As we will see in our numerical analysis later on this is a generic feature of the parameter space.

As the A-terms are crucial for the prediction of the physical Higgs mass, it is interesting to discuss the dependence on η_A in more detail. For the first solution this is trivial, while the second solution can be written in terms of η_1 and η_2 depending on η_A so that we find

$$\begin{aligned}
\eta_1 &= -8.0849 + 6.0757\eta_A \mp 4.9510\sqrt{(2.1354 - \eta_A)(\eta_A - 0.0962)}, \\
\eta_2 &= -3.0681 + 2.4288\eta_A \pm 0.8483\sqrt{(2.1354 - \eta_A)(\eta_A - 0.0962)},
\end{aligned} \tag{20}$$

which gives real ratios only for $0.0962 < \eta_A < 2.1354$. Interestingly, this implies for example that in our benchmark point with $\eta_A = -5$ the fine-tuning cannot be made arbitrarily small

by choosing appropriate η_1 and η_2 . Still the fine-tuning can be significantly reduced: for this benchmark point we find a minimal fine-tuning in the gaugino sector of

$$\Delta = 4.95 \left| \frac{M_3^2}{M_Z^2} \right| , \quad (21)$$

for $\eta_1 = -11.7$ and $\eta_2 = -4.6$ compared to $\Delta = 7.49 |M_3^2/M_Z^2|$ for $\eta_1 = \eta_2 = 1$.

Note that we have discussed here the fine-tuning in M_3 and η_A . As we mentioned above, there are regions of the parameter space where the fine-tuning in μ is dominant. Nevertheless, Δ_μ is also reduced by the choices of η_1 , η_2 and η_A we have just discussed. From Eq. (4) we see that making $|m_{h_u}^2|$ small reduces the fine-tuning in μ . For solution 1 this is obvious. In the case of solution 2, it is easy to check that the contribution to $|m_{h_u}^2|$ from the gaugino masses and trilinears is set to zero, too.

The first solution for the case $\eta_A = 0$ was already found in [7] - albeit with a slightly different notation setting $\eta_2 = 1$ instead of $\eta_3 = 1$ and thus giving hyperbolas instead of ellipses. The second solution, however, was not discussed therein. Instead, in [6] a related case was studied where also a fixed relation between the gaugino and the trilinear mass scales was assumed, while in our analysis η_A is a free parameter. We will discuss which GUT scale gaugino mass ratios are favoured, comparing numerical with semi-analytical results, in section 4.

3 GUT Scale Yukawa Ratios and SUSY Threshold Corrections

In GUTs quarks and leptons are unified in representations of the GUT symmetry group, which implies that the elements of the Yukawa matrices are generally related. As a consequence, fixed ratios of quark and lepton masses are typically predicted by GUT models. These ratios are given by group theoretical Clebsch–Gordan factors and hold at the GUT scale. For the third family, phenomenologically interesting possibilities are, e.g., the standard bottom-tau Yukawa unification $y_\tau/y_b = 1$ or the recently proposed ratio $y_\tau/y_b = 3/2$ [22]. The GUT scale Yukawa ratios are thus interesting discriminators between GUT models. We are therefore also interested in which GUT-scale Yukawa coupling relations might be preferred due to lower fine-tuning.

The phenomenological viability of GUT scale Yukawa ratios depends on SUSY threshold corrections [23]. Here we give some short comments on this important class of one-loop corrections to the Yukawa couplings of the down-type quarks and the charged leptons in the MSSM, which are sizeable if $\tan \beta$ is large. These corrections depend on the SUSY breaking parameters and hence the GUT scale Yukawa coupling ratios depend on the SUSY spectrum.

For example for the bottom quark mass we can write

$$m_b = y_b v_d (1 + \epsilon_b \tan \beta) , \quad (22)$$

the correction ϵ_b being given by [24–26]

$$\epsilon_b \approx \epsilon^G + \epsilon^B + \epsilon^W + \epsilon^Y , \quad (23)$$

where

$$\epsilon^G = -\frac{2\alpha_S}{3\pi} \frac{\mu}{M_3} H_2(u_{\tilde{Q}_3}, u_{\tilde{d}_3}) , \quad (24)$$

$$\epsilon^B = \frac{1}{16\pi^2} \left[\frac{g'^2}{6} \frac{\eta_1 M_3}{\mu} \left(H_2(v_{\tilde{Q}_3}, x_1) + 2H_2(v_{\tilde{d}_i}, x_1) \right) + \frac{g'^2}{9} \frac{\mu}{\eta_1 M_3} H_2(w_{\tilde{Q}_3}, w_{\tilde{d}_3}) \right] , \quad (25)$$

$$\epsilon^W = \frac{1}{16\pi^2} \frac{3g^2}{2} \frac{\eta_2 M_3}{\mu} H_2(v_{\tilde{Q}_3}, x_2) , \quad (26)$$

$$\epsilon^Y = -\frac{y_t^2}{16\pi^2} \frac{\eta_t M_3}{\mu} H_2(v_{\tilde{Q}_3}, v_{\tilde{u}_3}) . \quad (27)$$

Here $u_{\tilde{f}} = m_{\tilde{f}}^2/M_3^2$, $v_{\tilde{f}} = m_{\tilde{f}}^2/\mu^2$, $w_{\tilde{f}} = m_{\tilde{f}}^2/(\eta_1^2 M_3^2)$, $x_i = (\eta_i^2 M_3^2)/\mu^2$ for $i = 1, 2$ and the loop function H_2 reads

$$H_2(x, y) = \frac{x \ln x}{(1-x)(x-y)} + \frac{y \ln y}{(1-y)(y-x)} . \quad (28)$$

In the CMSSM one can neglect in a first approximation ϵ^B and ϵ^W because they are suppressed by the small gauge couplings. In NUGM scenarios this is in general not true anymore because the suppression might be compensated by an enhancement of the bino or wino mass parameter compared to the gluino one. In our example point we have seen that, for example, $\eta_1 = -11.7$ and $\eta_2 = -4.6$ is preferred to have low fine-tuning.

Without some knowledge of the parameter space or simplifying assumptions a quantitative statement is hence quite difficult. But in general one can expect significant corrections up to 50 % for the GUT scale Yukawa coupling ratios (see, e.g. [25, 26]).

4 Numerical Analysis

To improve our understanding based on the semi-analytical treatment above we now turn to our numerical analysis. To this end we employed a modified version of **softSUSY** [27] to calculate the SUSY spectra, GUT scale Yukawa couplings and the fine-tuning. The modifications take into account the implicit M_Z dependence on the Higgs vev [28] and the SUSY threshold corrections for the Yukawa couplings of all three fermion generations [23–26].

In order to compare with experimental constraints the observables $\text{BR}(b \rightarrow s\gamma)$, $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ and $\text{BR}(B_u \rightarrow \tau\nu_\tau)$ were calculated using **SuperIso** [29]. The experimental ranges we considered for such observables are: $\text{BR}(b \rightarrow s\gamma) = (355 \pm 24 \pm 9)10^{-6}$ [30], $\text{BR}(B_s \rightarrow \mu^+\mu^-) < 4.5 \cdot 10^{-9}$ at 95% CL [16] and $\text{BR}(B_u \rightarrow \tau\nu_\tau) = 1.41 \pm 0.43$ [31]. Additionally, LEP bounds [32] on sparticle masses were applied and we discard points corresponding to a $\tilde{\tau}$ LSP and color and charge breaking (CCB) vacua. We present results with and without the Higgs boson mass constraint included to study its impact. Applying constraints from direct LHC SUSY searches is not straightforward and we will only show a comparison to simplified models for illustration.

In our opinion the 3.2σ level $(g-2)_\mu$ discrepancy from the Standard Model expectation [11] is an experimental evidence that, while certainly interesting, still requires a full experimental confirmation from next generation experiments, such as those proposed at J-PARC [33] and at Fermilab [34]. For this reason we have not imposed constraints from the anomalous magnetic moment of the muon. In fact in this scenario we can only find very few and isolated points

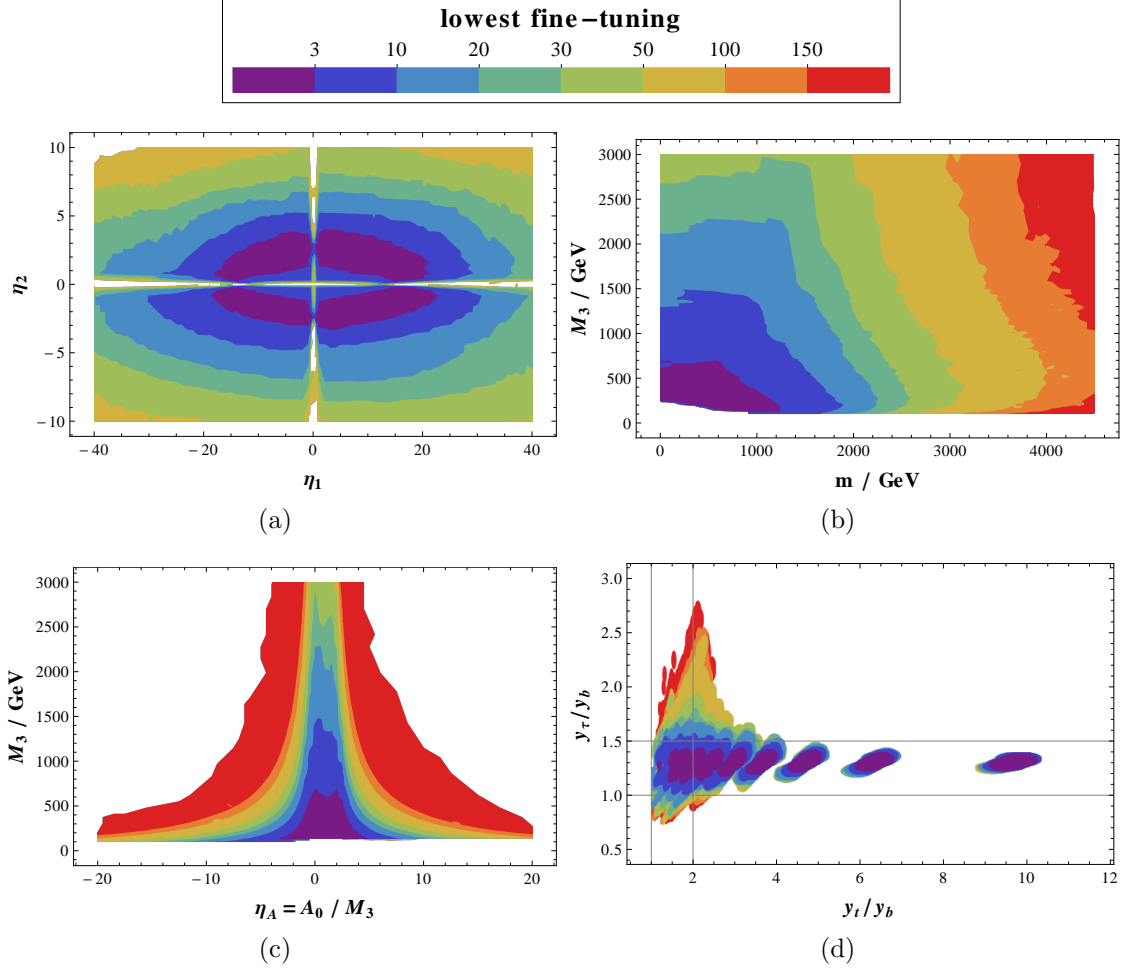


Figure 2: Lowest fine-tuning in the (a) η_1 - η_2 , (b) m - M_3 , (c) η_A - M_3 and (d) y_t/y_b - y_τ/y_b planes consistent with the experimental bounds described in the text. For explanations of the parameters see main text. The Higgs mass constraint is not included. The 1σ errors on the quark masses [32] are taken into account by scaling the data points in the last plot correspondingly.

which would satisfy $(g-2)_\mu$ at 2σ and predict a Higgs boson mass in agreement with the recent results, all of them corresponding to a fine-tuning larger than $\mathcal{O}(100)$.

Moreover, we have not imposed constraints from neutralino relic density nor from direct and indirect dark matter searches. Such constraints can be evaded assuming a non-standard cosmological history or a different LSP, like the gravitino or the axino. Nevertheless, it is interesting to note that for $\eta_2 \lesssim 0.5 \eta_1$, the lightest neutralino is dominated by its Wino (or Higgsino) component, which implies that the relic density is strongly suppressed. In these regions, on the one hand, one cannot explain dark matter by thermal relic neutralinos, but, on the other hand, there is no overproduction of neutralino dark matter.

4.1 Before the latest LHC Higgs and SUSY Results

As previously discussed, we expect to find an ellipse shaped region in the η_1 - η_2 plane where fine-tuning is significantly lower than in other parts of the parameter space, especially when compared to the CMSSM. To study this in detail, we scanned the NUGM parameter space in the following region: the soft scalar mass parameters were assumed to be universal $m_i = m$ for every i and varied from 0 to 4.5 TeV, the GUT scale gluino mass M_3 from 0.15 to 2 TeV, and $\eta_t = \eta_b = \eta_\tau = \eta_A$ from -20 to 20 for $\tan\beta = 2, 10, 15, 20, \dots, 60$. For the ranges of the gaugino mass ratios we took $-40 \leq \eta_1 \leq 45$ and $-10 \leq \eta_2 \leq 10$. Both signs for μ were allowed. In the scan we have dropped all points with a fine-tuning larger than 200.

Fine-tuning was calculated for the parameters m , μ , $A_0 = \eta_A M_3$ and M_3 . Nominally existing tuning in the ratios η_1 , η_2 was neglected as we assume such relations to be rigid and given by some underlying model. We note as a side remark that in all plots where contours are shown instead of only points, outliers that are very far away from their neighbours are still shown as isolated points instead of being incorporated into the contour. In addition, unless stated otherwise, all plots show values marginalised over parameters not shown.

As we can see from Fig. 2a, we indeed find an ellipse that allows for quite low fine-tuning and evades the experimental bounds. We see that the semi-analytical results from Fig. 1 do not very well reproduce the numerical results but still they give a good qualitative understanding. Compared to the CMSSM with the same experimental bounds, we find a fine-tuning Δ lower than 3 instead of just below 10 [5]. In this section, we will focus on this region of low fine-tuning and only consider points with an (η_1, η_2) combination lying on the ellipse defined by $\Delta_{\min} < 10$.¹⁰ An approximate analytical formula for the ellipse in Fig. 2a is given by

$$(\eta_1/15.0)^2 + (\eta_2/2.6)^2 = 1. \quad (29)$$

Based on this, we take a closer look at the fine-tuning dependence on the other parameters. In Fig. 2b we can clearly see that introducing non-universal gaugino masses can significantly weaken the dependence of Δ_{\min} on the gaugino mass scale compared to the CMSSM. While in the latter a fine-tuning of 20 is only possible for $M_3 < 400$ GeV [5], now we can go up to $M_3 \sim 2.2$ TeV. On the other hand, the fine-tuning dependence on m (resp. m_0 in the CMSSM) is very much the same as expected.

Furthermore from Fig. 2c we can roughly see the same behaviour for large η_A as in the CMSSM [5], while for small η_A from -2 to 4 we find two peaks. They lie at $\eta_A = 0$ and $\eta_A \sim 2$, which are the values for η_A required for solution 1 and the upper end of the range quoted for solution 2, cf. Eqs. (18, 19).

Finally, concerning the GUT scale Yukawa coupling ratios shown in Fig. 2d we find no qualitative difference compared to the CMSSM, besides the rather obvious fact that the fine-tuning price of all ratios is greatly reduced. The comparison of the b - τ Yukawa coupling ratio of $3/2$ with the case of b - τ Yukawa unification, however, is slightly in favor of $3/2$ with a minimal fine-tuning of $\Delta \sim 5$ vs. 9.

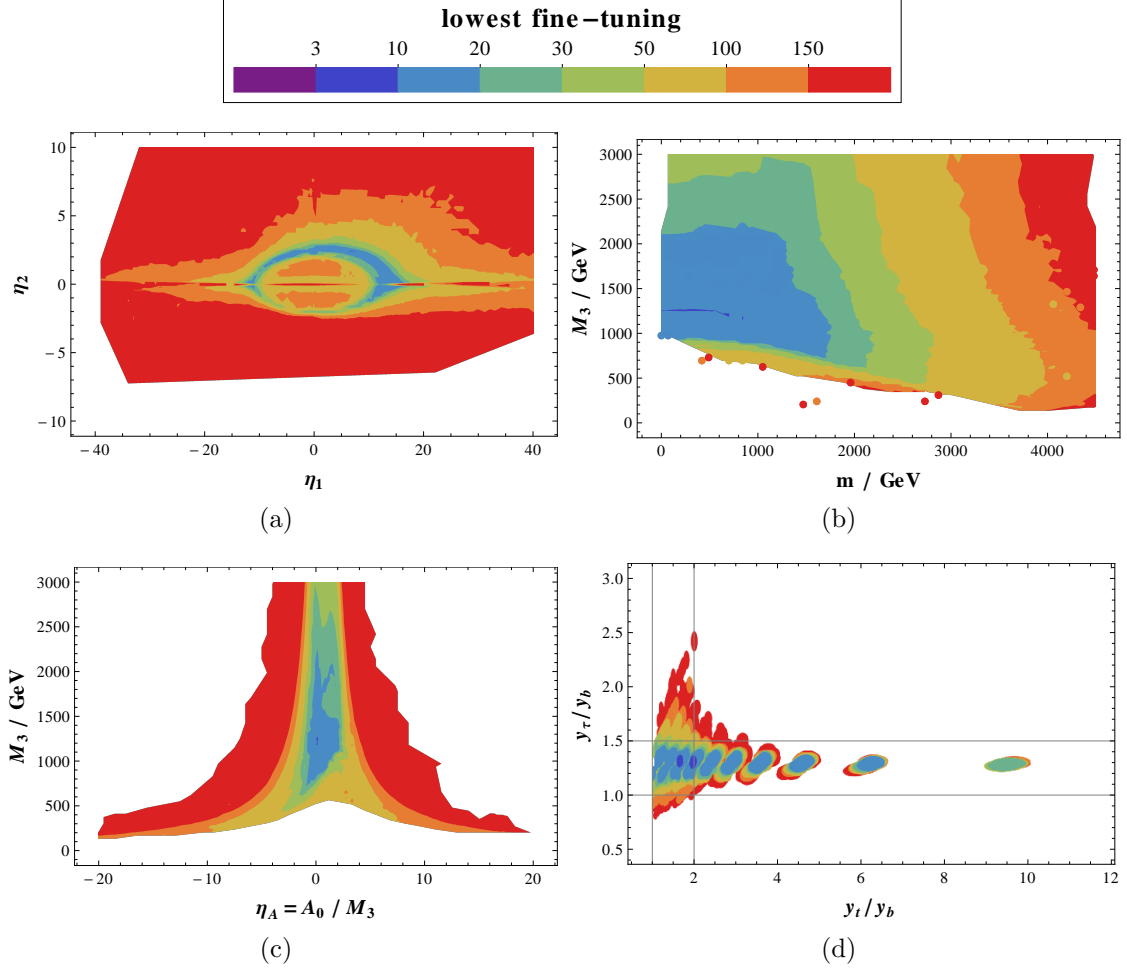


Figure 3: Lowest fine-tuning in the (a) η_1 - η_2 , (b) m - M_3 , (c) η_A - M_3 and (d) y_t/y_b - y_τ/y_b planes consistent with the experimental bounds described in the text. For explanations of the parameters see main text. In comparison to Fig. 2 we have included the CMS experimental constraint $m_h = 125.3 \pm 0.6$ GeV [1] and a theoretical uncertainty of ± 3 GeV [38] for the Higgs mass calculation at each data point. The 1σ errors on the quark masses [32] are taken into account by scaling the data points in the last plot correspondingly.

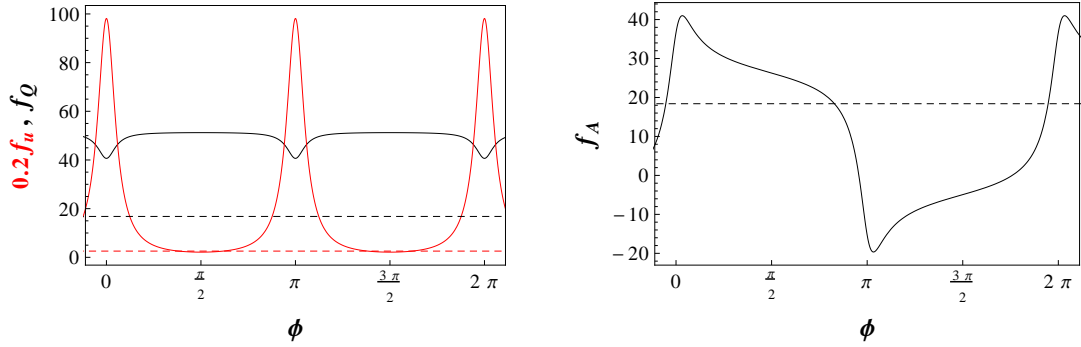


Figure 4: Dependence of gaugino mass contribution functions f_Q , f_u , f_A (left plot/black, left plot/red, right plot respectively) to left-handed stop soft squared mass $m_{\tilde{Q}}^2$, right-handed stop soft squared mass $m_{\tilde{u}}^2$ and stop trilinear coupling A_t as defined in Eq. 30. For the ellipse of low fine-tuning the rough form of Eq. (29) was used and we have defined $\phi = \arg(\eta_1 + i\eta_2)$. The corresponding value of f_x for $\eta_1 = \eta_2 = 1$ is shown as dashed line in the same color. Note that f_u has been rescaled by a factor 5 for illustration.

4.2 Results including the latest LHC Higgs and SUSY Searches

In the light of the discovery of a possible $125 \div 126$ GeV Higgs boson in current LHC searches, it is interesting to re-analyse the consequences of non-universal gaugino masses for naturalness of the (non-universal) MSSM. In Fig. 3 we show the same plots as in Fig. 2 with the additional experimental constraint from CMS, $m_h = 125.3 \pm 0.6$ GeV [1], on top of which we include a theoretical uncertainty of ± 3 GeV [38] for the Higgs mass calculation at each data point.

As we can see from Fig. 3a, non-universal gaugino masses can accommodate this additional constraint even with a fine-tuning lower than 20 (actually just above 10 and in a very small region of parameter space even slightly below 10). This happens e.g., around $\eta_2 \sim 0$, $\eta_1 \sim 15$ and to a lesser degree also around $\eta_1 \sim 0$, $\eta_2 \sim 2.6$ and in the intermediate part of the ellipse. The reasons why these regions are favoured after including the Higgs results can readily be seen from the beta functions [15] of the stop soft masses masses and the stop trilinear coupling at the GUT scale:

$$16\pi^2 \beta_{m_{\tilde{Q}^2}} \supset -g_{\text{GUT}}^2 M_3^2 \left(\frac{32}{3} + 6\eta_2^2 + \frac{2}{15}\eta_1^2 \right) \equiv -g_{\text{GUT}}^2 M_3^2 f_Q(\eta_1, \eta_2), \quad (30a)$$

$$16\pi^2 \beta_{m_{\tilde{u}^2}} \supset -g_{\text{GUT}}^2 M_3^2 \left(\frac{32}{3} + \frac{2}{15}\eta_1^2 \right) \equiv -g_{\text{GUT}}^2 M_3^2 f_u(\eta_1, \eta_2), \quad (30b)$$

$$16\pi^2 \beta_{A_t} \supset -g_{\text{GUT}}^2 M_3 \left(\frac{32}{3} + 6\eta_2 + \frac{26}{15}\eta_1 \right) \equiv -g_{\text{GUT}}^2 M_3 f_A(\eta_1, \eta_2). \quad (30c)$$

The functional behaviour of f_Q , f_u and f_A with $\eta_1 = r_\eta \cos \phi$, $\eta_2 = r_\eta \sin \phi$ and by means of the approximate form of the numerically obtained ellipse of Eq. (29), is shown in Fig. 4. It can be easily seen that the gaugino mass contribution to right-handed stop masses is greatly enhanced for the angles $\phi = 0, \pi$, i.e. $\eta_2 = 0$, $|\eta_1| \sim 15$, while it is not significantly away from the CMSSM value elsewhere. The contribution to the left-handed stop mass on the other

¹⁰Depending on the other parameters these points can have a larger fine-tuning but, interestingly, this does not significantly change the appearance of all affected plots, indicating that being on the ellipse is indeed a necessary condition for small fine-tuning.

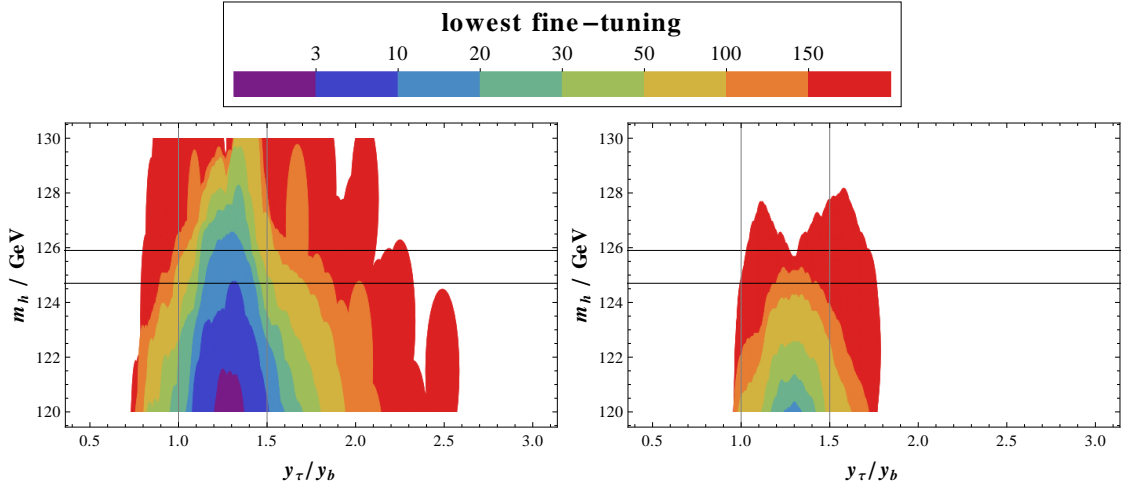


Figure 5: Lowest fine-tuning in the m_h - y_τ/y_b plane consistent with experimental bounds described in the text for non-universal gaugino masses (left) and universal gaugino masses (right). The horizontal lines correspond to the 1σ error of 0.6 GeV around 125.3 GeV as claimed by the CMS collaboration [1]. The 1σ errors on the quark masses [32] are taken into account by scaling the data points correspondingly. In addition, a theoretical uncertainty of 3 GeV [38] for the Higgs mass calculation at each data point is taken into account.

hand receives a smaller but still sizable enhancement over the CMSSM value, but does not vary much along the ellipse. Additionally for $\phi = 0$, i.e. $\eta_1 \sim 15$ and $\eta_2 = 0$, the running contribution to A_t receives a sizeable enhancement, which is present to a lesser degree also for $\phi = \pi/2$, i.e. $\eta_1 = 0$ and $\eta_2 \sim 2.6$. For angles in the other quadrants, where at least one η_i is negative, however, the cancellation with the large gluino mass running contribution prevents from a similar enhancement in f_A . However, in practice, these three effects turn out to be effective only when they work in unison to obtain a high enough Higgs mass.

We note that the changes in Fig. 3b are not very surprising: regions with low masses get cut off. Also the changes in Fig. 3c are not unexpected, the solution with $\eta_A > 0$ is disfavoured more than the solution with $\eta_A \sim 0$, as the latter does not suffer from the cancellation with the gluino mass contribution to the running of the top trilinear soft term. Unsurprisingly, it is shown that either large η_A or large M_3 (hence large top trilinear coupling) is needed to accommodate a large Higgs mass.

Concerning the Yukawa coupling ratios shown in Fig. 3d we can see that the minimal fine-tuning required to get to a unified b - τ Yukawa coupling ratio suffers more from the requirement of a consistent Higgs mass than the ratio of $3/2$ does. Namely after applying the cut the fine-tuning for $y_\tau/y_b = 3/2$ can go down to $\Delta = 30$, while $y_\tau = y_b$ requires at least a Δ of 60. This is also nicely illustrated in Fig. 5 where we have shown the interplay among the Higgs mass, the GUT scale y_τ/y_b ratio and the amount of fine-tuning.

Another important aspect of current LHC experiments are the searches for supersymmetric particles. Since changes of the gaugino mass ratios η_1 and η_2 can significantly alter the composition of the lightest neutralino¹¹ as well as the mass splittings controlling jet energies and missing E_T from the cascade decays, the exclusion regions for the CMSSM do not apply

¹¹Actually, low fine-tuning generally also means quite low μ , so Higgsino-like lightest neutralinos and charginos are likely.

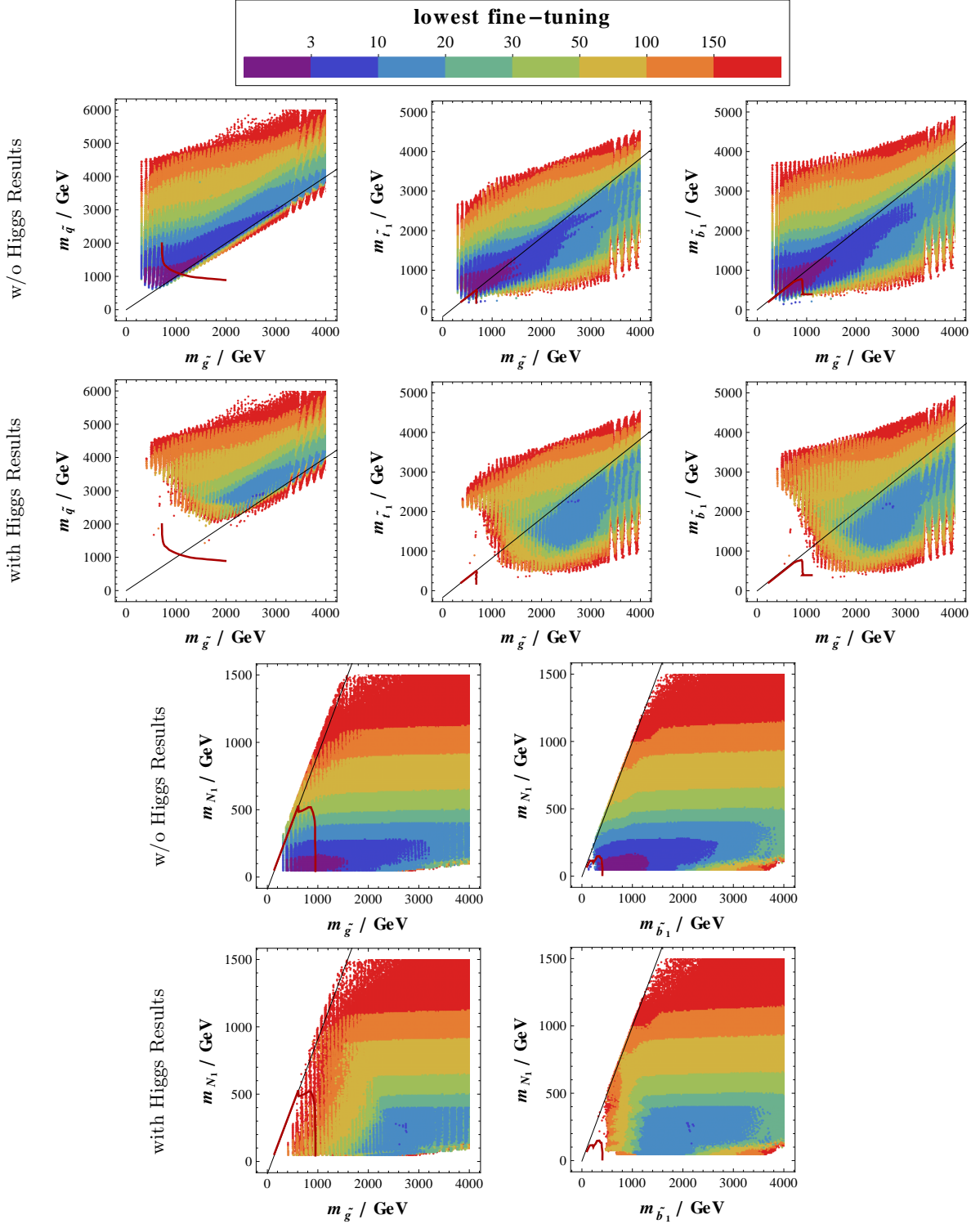


Figure 6: Lowest fine-tuning in various planes used for simplified models. Only points consistent with experimental bounds described in the text are shown. In addition, for the plots on the second and fourth line we have included the experimental constraint $m_h = 125.3 \pm 0.6$ GeV [1] and a theoretical uncertainty of ± 3 GeV [38] for the Higgs mass calculation at each data point. The corresponding bounds due to LHC SUSY searches are shown as thick dark red lines [4].

η_1, η_2	Δ_{\min}	Origin
1, 1	118	CMSSM (Gaugino Unification)
10, 2	12	200 of $SU(5)$ [37]
$\frac{19}{10}, \frac{5}{2}$	18	770 of $SO(10) \rightarrow (1, 1)$ of $SU(4) \times SU(2)_R$ [37]
$\frac{77}{5}, 1$	36	770 of $SO(10) \rightarrow (1, 0)$ of $(SU(5)' \times U(1))_{\text{flipped}}$ [37]
$-\frac{1}{5}, 3$	46	210 of $SO(10) \rightarrow (75, 0)$ of $(SU(5)' \times U(1))_{\text{flipped}}$ [37]
$\frac{21}{5}, \frac{7}{3}$	13	O-II with $\delta_{\text{GS}} = -6$ [7, 39]
$\frac{17}{5}, 2$	28	O-II with $\delta_{\text{GS}} = -7$ [7, 39]
$\frac{29}{5}, 3$	44	O-II with $\delta_{\text{GS}} = -5$ [7, 39]

Table 1: Selected ratios and the minimal possible fine-tuning they allow after requiring the experimental constraint $m_h = 125.3 \pm 0.6$ GeV [1] and a theoretical uncertainty of ± 3 GeV [38] for the Higgs mass calculation. Only ratios that can reduce the fine-tuning by at least 50% compared to the unified (CMSSM) scenario are shown. For more details on the origin of these ratios, see, e.g. [7, 37, 39]. The results are illustrated graphically in Fig. 8.

anymore. A full event and detector simulation would, however, go beyond the scope of this study; thus we make use of exclusion bounds derived in so-called simplified models [4, 36]. For simplicity we just compare the spectra found by the numerical scan with the most stringent bounds in several kinds of simplified models. While this is not certainly a rigorous approach, it should give a good feeling how endangered by exclusion each point is.

The resulting situation is shown in Fig. 6. As we can see in most cases only the region with $\Delta < 3$ is partially inside the excluded region, while even $\Delta < 10$ extends far beyond the bounds. Requiring a Higgs boson within 1σ of the experimental measurement [1] (including 3 GeV theoretical uncertainty as explained before) excludes even more than what direct SUSY searches do in some areas. It is thus important to note that, even in this quite restrictive approach, all of the parameter space with high m_h and $\Delta < 20$ is safe from being excluded by the LHC in the near future.

For future searches it is also interesting to look at the correlation between the Higgs and several sparticle masses, which is shown in Fig. 7. There we can see that for low fine-tuning relatively light neutralinos and charginos are expected. This is due to the fact that μ is small in this region, so that we expect Higgsinos to be light. We also see that the gluinos and the lightest stop are rather heavy in our scenario with low fine-tuning ($\Delta < 20$ for $m_{\tilde{g}} \gtrsim 1.5$ TeV and $m_{\tilde{t}_1} \gtrsim 1.0$ TeV). The LHC still did not reach this parameter region and hence the statement that the natural MSSM parameter space is already ruled out seems to be premature.

4.3 Favoured Non-Universal Gaugino Mass Ratios

Fixed non-universal gaugino mass ratios may originate from various high energy models, for instance from GUTs or orbifold scenarios (see, e.g. [7, 37, 39] for discussions). Tab. 1 and Fig. 8 show examples of proposed fixed ratios η_1, η_2 where we find that the fine-tuning can be reduced by more than 50% compared to the CMSSM.

Interestingly, among the orbifold models O-II of [7, 39], from the full numerical results

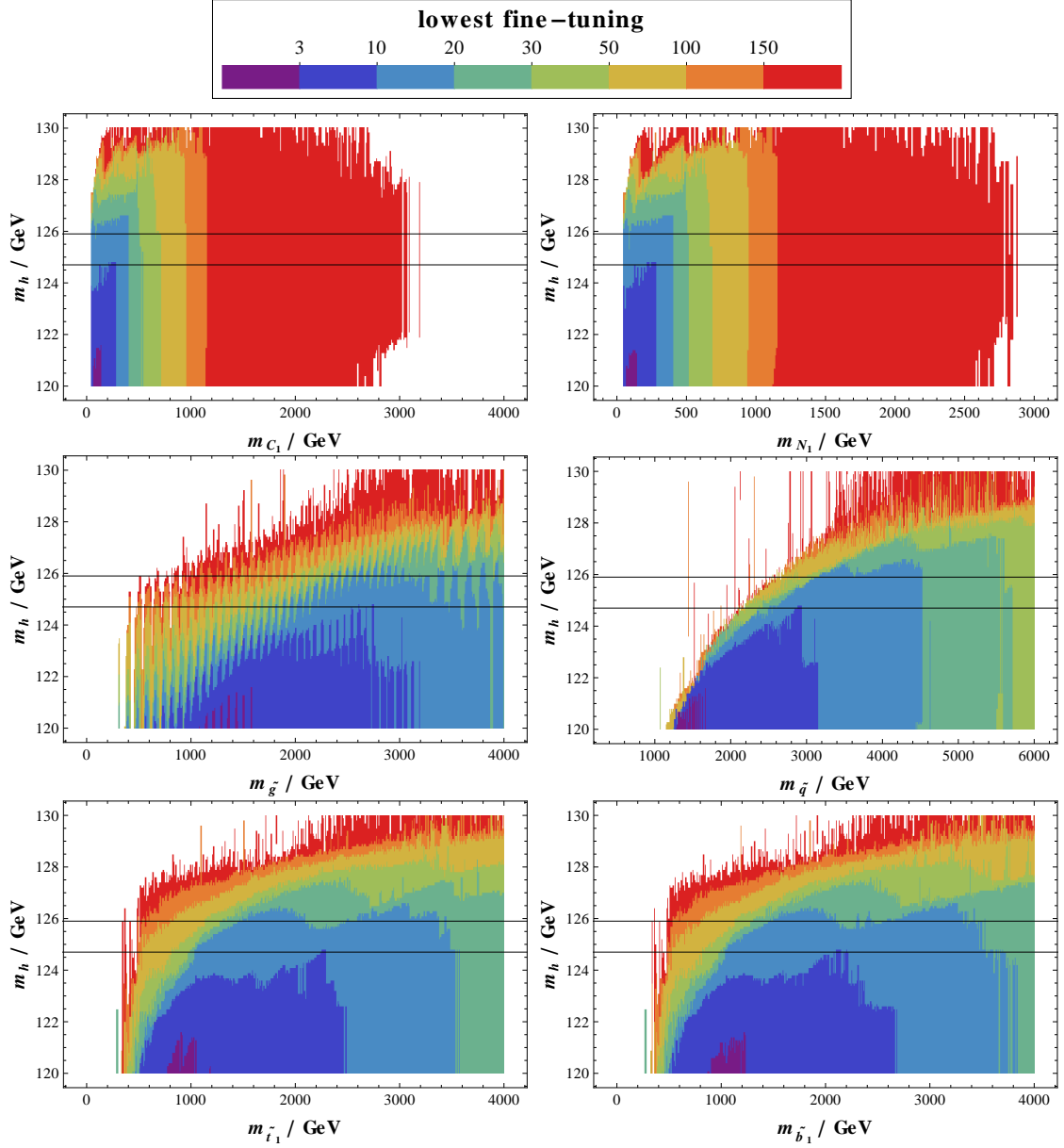


Figure 7: Lowest fine-tuning shown for the Higgs mass vs. important particle masses including the experimental constraint $m_h = 125.3 \pm 0.6$ GeV [1] and a theoretical uncertainty of ± 3 GeV [38] for the Higgs mass calculation at each data point.

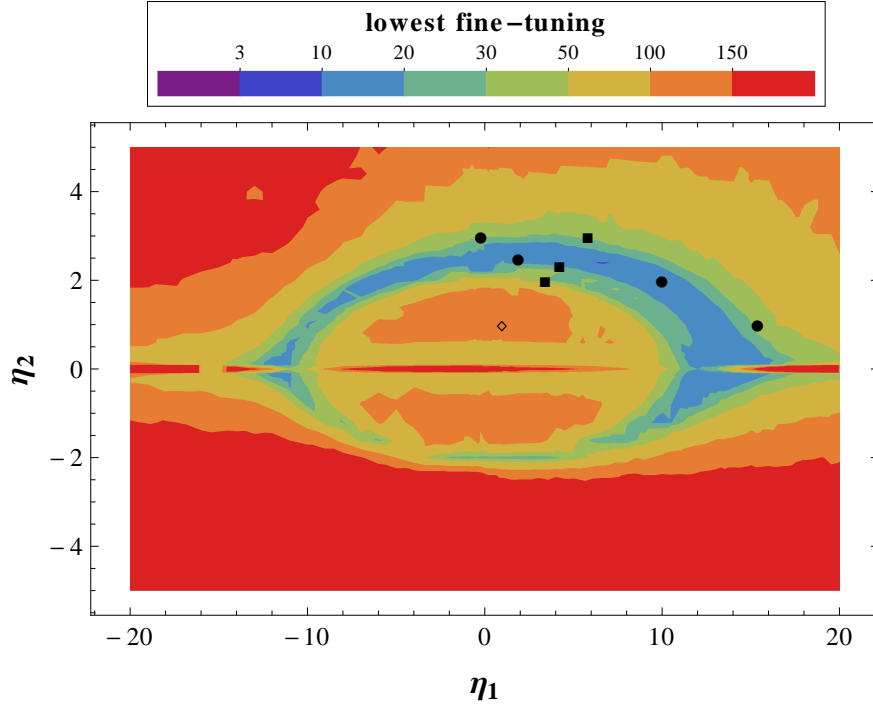


Figure 8: Lowest fine-tuning in the η_1 - η_2 plane consistent the experimental constraint $m_h = 125.3 \pm 0.6$ GeV [1] and a theoretical uncertainty of ± 3 GeV [38] for the Higgs mass calculation at each data point. Theoretically motivated ratios that reduce the fine-tuning compared to the unified scenario by at least 50% and the analytical expectation are also included. The CMSSM ratio is marked as empty diamond, circles correspond to ratios derived from GUT symmetry breaking [37] and squares to ratios found in the so called O-II model in [7, 39]. For details see Tab. 1.

including the constraints from the latest Higgs results we find that the option $(\eta_1, \eta_2) = (\frac{21}{5}, \frac{7}{3})$ from $\delta_{\text{GS}} = -6$ has the lowest possible fine-tuning ($\Delta_{\text{min}} = 13$), whereas in [7], based on analytical estimates before the Higgs results were available, the preferred ratio was $(\eta_1, \eta_2) = (\frac{29}{5}, 3)$ from $\delta_{\text{GS}} = -5$.¹²

The GUT ratio with lowest fine tuning ($\Delta_{\text{min}} = 12$) turned out to be $(\eta_1, \eta_2) = (10, 2)$. For the ratios found to be favoured in [7], we find significantly higher fine-tuning, e.g.: $\Delta_{\text{min}} = 82$ for $(\eta_1, \eta_2) = (-5, 3)$, $\Delta_{\text{min}} = 141$ for $(\eta_1, \eta_2) = (-\frac{101}{10}, -\frac{3}{2})$, $\Delta_{\text{min}} = 143$ for $(\eta_1, \eta_2) = (1, -\frac{7}{3})$.

As it can be seen as well from comparison of Fig. 2a and 3a, the inclusion of the Higgs results has strong effects on the favoured non-universal gaugino mass ratios.

Finally, as discussed at the beginning of section 4, for $\eta_2 < 0.5 \eta_1$ the neutralino is dominated by its Wino (or Higgsino) component which implies that the relic density is strongly suppressed and there is therefore no danger of overproducing thermal neutralino dark matter. Among the ratios listed in Tab. 1 this applies to $(\eta_1, \eta_2) = (10, 2)$ and $(\eta_1, \eta_2) = (\frac{77}{5}, 1)$. For the other ratios of Tab. 1, $\eta_2 > 0.5 \eta_1$ holds and, at least in principle, thermal neutralino dark matter could be possible (for a standard thermal history of the universe). As already mentioned, we have not applied dark matter constraints in our analysis since they depend on the thermal history of the universe.

5 Summary and Conclusions

We have re-addressed the question of the naturalness of the MSSM, in the light of the discovery of a Higgs-like resonance recently announced by the LHC experiments [1]. We focused on models with non-universal boundary conditions of the SUSY breaking parameters at high energy and compared them with the CMSSM. Our basic assumption was that the departures from universality are a consequence of an underlying mechanism, possibly associated to SUSY breaking or GUT dynamics, giving fixed relations among the parameters. In order to identify which of such relations can lead to a reduced fine-tuning, we first considered a general high-energy parametrization of the soft terms. Then we discussed scenarios with non-universal scalar and gaugino masses. We found this latter possibility particularly promising (as already noticed, e.g., in [6, 7]) and we thus studied it in detail, computing numerically the fine-tuning measure Δ .

We found that, considering the uncertainty related to the theoretical prediction, models with non-universal gaugino masses can account for a Higgs mass in the CMS 1σ range 125.3 ± 0.6 with a fine-tuning price of $\mathcal{O}(10)$, in contrast to the CMSSM that requires $\Delta_{\text{min}} \gtrsim \mathcal{O}(100)$. Thus the MSSM with specific gaugino mass ratios still represents a comparatively natural scenario. In Fig. 2 and 3, we have shown the values of $\eta_1 = M_1/M_3$ and $\eta_2 = M_2/M_3$ giving a low fine-tuning before and after applying a constraint on the Higgs mass. Interestingly, some of the ratios discussed in the literature lie in (or are close to) the new low fine-tuning region, while others are disfavoured when the new Higgs results are included. Including the Higgs results we found that particularly favoured ratios (with $\Delta_{\text{min}} = \mathcal{O}(10)$) are now, e.g., $(\eta_1, \eta_2) = (10, 2)$ which may originate from $SU(5)$ GUTs and $(\eta_1, \eta_2) = (\frac{21}{5}, \frac{7}{3})$ from orbifold scenarios of type O-II with $\delta_{\text{GS}} = -6$.

Furthermore, allowing for non-universal gaugino masses at the GUT scale, we have analyzed the fine-tuning price of different values of the GUT-scale ratio y_τ/y_b , that represents an important handle to discriminate among different GUT models. For $m_h \approx 125$ GeV, we found

¹² δ_{GS} is a negative integer constant associated with Green-Schwarz anomaly cancellation (cf. [7, 39]).

that b - τ Yukawa unification corresponds to $\Delta \gtrsim 60$, while the alternative ratio $y_\tau/y_b = 3/2$ can be realized at the price of $\Delta \gtrsim 30$ only.

Concerning the SUSY spectrum that is favoured by naturalness and $m_h \approx 125$ GeV, we found that for the least tuned data points with fine-tuning Δ less than 20 the lightest neutralino is expected to be lighter than about 400 GeV and the lighter stop can be as heavy as 3.5 TeV. On the other hand, the gluino mass is required to be above 1.5 TeV. Comparing the predicted spectra with the LHC exclusions derived in a set of simplified models, we could conclude that the regions of lowest fine-tuning are at present only poorly constrained by direct SUSY searches at the LHC.

Let us finally remark that, although the CMSSM is certainly challenged from the fine-tuning point of view by a Higgs mass of $m_h \approx 125$ GeV, more general realizations of the MSSM, like the examples studied here, can still provide a relatively natural solution of the hierarchy problem and will probably require several years of data taking before being fully tested at the LHC.

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